Green's function analysis of electromagnetic waves in two-layered periodic structures with fluctuations in thickness

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A general method for the construction of the Green's function for finite one-dimensional inhomogeneous layers is developed. Using the results of this method the exact analytical Green's function for periodic dielectric structures is found. As an example of its application, the influence of fluctuations of the widths of the basic layers on the reflection and transmission of electromagnetic waves propagating through the structure is investigated. The results are applied to the design of optical switching systems with periodic dielectric structures as the operating medium. The same Green's function can be used to solve a wide variety of other problems.

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I. INTRODUCTION

In a previous article [1] we considered in detail the normal propagation of plane electromagnetic waves through two-layered periodic dielectric structures. We found that even relatively small variations in the parameters of the structure could cause large changes in the reflection and transmission. As an example, we demonstrated the influence of a constant elastic stress created inside the structure on the reflection coefficient. We showed that in some regions of the structure parameters, particularly near the boundaries between so-called forbidden and allowed regions of wave frequencies, a reasonably small stress is capable of shifting an electromagnetic wave with wavelength λ from a forbidden region, where reflection is close to 100%, to the edge of the allowed region, where reflection is not more than 15% to 20%. Such shifts are caused primarily by homogeneous changes, i.e., changes that keep the periodicity of the structure, with the width of the basic layers changing from d_1, d_2 to $d_1+\delta d_1, d_2+\delta d_2$, with δd_1 and δd_2 being around 1.5% for materials like polystyrene. The aforementioned shifting opens the possibility of using a two-layered periodic dielectric structure as the operating medium for optical devices like modulators and switches. However, there may be practical difficulties in achieving this purpose due to random fluctuations in any real structure. For us the most important issue of this ''nonideality'' is random fluctuations in thickness of the layers due to inhomogeneous growing conditions. As a result, the real reflection and transmission coefficients could have features that differ considerably from those for ideal structures. Despite intense theoretical and experimental investigation of layered periodic structures in recent years [2–8], including nonlinear cases [9,10], the influence of possible defects on the properties of reflection and transmission coefficients of these structures are much less well known; see Refs [11,12], which are mainly devoted to the description of new pseudogaps of localized states created by positional disorder. Therefore, a theory that is able to take into account the effect of such defects (fluctuations in layer thickness in our case) on the reflection and transmission is needed.

The most general and perhaps the most elegant approach to such problems is to find the Green's function for the unperturbed profile of the refractive index, which in our case is an ideal two-layered periodic structure. Then, using the Green's function, it is possible to reduce the perturbed problem to the integral Lippmann-Schwinger equation and to find its solution, at least in the first and second approximations, in terms of the Neumann (Born) series.

In the present paper, we develop a general method for the construction of an analytical Green's function for the waves propagating normal to the surface of a finite one-dimensional structure with an arbitrary profile of refractive index, surrounded by a homogeneous medium with refractive index n_0. Using the Green's function obtained, we transform the differential wave equation to the integral Lippmann-Schwinger equation, taking into account outgoing scattered-wave boundary conditions. As an application of the theory developed, we consider the propagation of waves in two-layered periodic dielectric structures with fluctuations in the layer thickness. In particular, we obtain the exact analytical Green's function for the ideal structure (no fluctuations). Then, following the standard procedure of solving the Lippmann-Schwinger equation in terms of Neumann series, we obtain the first-order correction to the reflection coefficient for the ideal structure. After that, we define the limits of our fluctuations under which the Neumann series remains convergent and identify the regions of structure parameters where the reflection coefficient is not critically sensitive to the fluctuations, i.e. keeps the essential features of the ideal structure. Finally, combining the results of the analysis concerning the tolerance in fluctuations with our previous results concerning shifts of the electromagnetic wave from forbidden to allowed regions under elastic stress action [1], we suggest theoretical guidelines for the construction of optical switching systems with a two-layered periodic dielectric structure as an operating principle.

The paper is organized as follows. In Sec. II we develop a general method for the construction of an exact analytical form of the Green's function for one-dimensional systems with symmetrical refractive indexes at large distances and show how to make practical use of this function for the calculation of the reflection coefficients. In Sec. III we apply the results to the calculation of the reflection coefficient of a two-layered periodic structure with fluctuations in the layer thicknesses. In Sec. IV we identify the structure parameters,

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including the limits of fluctuations in them that are tolerable for optical switching systems. Finally, the conclusions are summarized in Sec. V.

II. GENERAL THEORY

Let us consider a transparent (without absorption) dielectric structure of length L with a position-dependent refractive index $n_L(z)$, and with a constant index of refraction n_0 of the medium on either side of the structure. Mathematically, the refractive index $n_h(z)$ of such a profile can be represented in the form

$$n_h(z)=\begin{cases} n_0, & z<0, z>L \\ n_L(z), & 0<z<L. \end{cases} \tag{1}$$

Let electromagnetic waves propagate along the z direction, corresponding to normal propagation. In this case, light polarization does not play a role and the calculations given below are valid for both possible wave polarizations (along the y or x axis). For monochromatic waves, i.e., for harmonic time dependence, we can set $E(z,t)=E(z)\exp(-i\omega t)$, and reduce the Maxwell equations to the one-dimensional homogeneous Helmholtz equation

$$\frac{d^2E(z)}{dz^2}+k^2n_h(z)^2E(z)=0, \tag{2}$$

where k is the wave number in vacuum.

The Green's function $G(z,z_1)$ for Eq. (2) satisfies the equation

$$\frac{d^2G(z,z_1)}{dz^2}+k^2n_h(z)^2G(z,z_1)=\delta(z-z_1) \tag{3}$$

and the radiation condition

$$G(\pm\infty,z_1)\sim\exp(\pm ikn_0z) \tag{4}$$

for the waves that fall on the structure from the region $z<0$. The general method of obtaining the solution for the Green's function $G(z,z_1)$ involves its spectral decomposition in terms of the normalized solutions of the homogeneous equation (2) (see, for example, Ref. [13]). But the calculations in an exact analytical form appear lengthy and cumbersome even for simple cases where the profile $n_L(z)$ is either a step [14] or quadratic function [15].

The first basic idea of the proposed method for the construction of an analytical Green's function associated with Eq. (2) is to use the symmetry property $G(z,z_1)=G(z_1,z)$. We divide the whole plane (z,z_1) into 12 parts, symmetrical in pairs with respect to the line $z=z_1$, and introduce the following notation for $G(z,z_1)$ in each of these parts:

$$G(z,z_1)=\begin{cases} G_1(z,z_1) & \text{if } z_1>L, \quad z<0 \\ G_2(z,z_1) & \text{if } 0<z_1<L, \quad z<0 \\ G_3(z,z_1) & \text{if } z<z_1<L, \quad z<0 \\ G_4(z,z_1) & \text{if } z_1<z, \quad z<0 \\ G_5(z,z_1) & \text{if } z_1>L, \quad 0<z<L \\ G_6(z,z_1) & \text{if } z<z_1<L, \quad 0<z<L \\ G_7(z,z_1) & \text{if } 0<z_1<z, \quad 0<z<L \\ G_8(z,z_1) & \text{if } z_1<0, \quad 0<z<L \\ G_9(z,z_1) & \text{if } z_1>z, \quad z>L \\ G_{10}(z,z_1) & \text{if } z_1<z, \quad z>L \\ G_{11}(z,z_1) & \text{if } 0<z_1<L, \quad z>L \\ G_{12}(z,z_1) & \text{if } z_1<0, \quad z>L \end{cases} \tag{5}$$

The symmetry properties of the Green's function require a knowledge of the solution only in the upper half plane (z,z_1) where $z_1>z$. We can obtain the solution in the lower half plane using the symmetry equalities $G_1(z,z_1)=G_{12}(z_1,z)$, $G_2(z,z_1)=G_8(z_1,z)$, $G_3(z,z_1)=G_4(z_1,z)$, $G_5(z,z_1)=G_{11}(z_1,z)$, $G_6(z,z_1)=G_7(z_1,z)$, and $G_9(z,z_1)=G_{10}(z_1,z)$.

Now let us derive the exact expressions for the Green's function in the upper half plane. First, we note that since $n(z)$ varies only along the z axis the Green's function $G(z,z_1)$ can be factorized into independent functions of z and z_1. Then, using the boundary conditions on the internal boundaries of the upper half plane and the boundary conditions along the line $z=z_1$, we can show that the exact analytical expressions for the function $G(z,z_1)$ in the upper half plane can be cast in the form

$$G_1(z,z_1)=\frac{1}{2ikn_0}B_l\exp[ikn_0(z_1-L)]\exp(-ikn_0z),$$

$$G_2(z,z_1)=\frac{1}{2ikn_0}[C_lE^L_{h_1}(z_1)+D_lE^L_{h_2}(z_1)]\exp(-ikn_0z),$$

$$G_3(z,z_1)=\frac{1}{2ikn_0}[\exp(ikn_0z_1)+A_l\exp(-ikn_0z_1)]\times\exp(-ikn_0z), \tag{6}$$

$$G_5(z,z_1)=\frac{1}{2ikn_0}\exp[ikn_0(z_1-L)][C_rE^L_{h_1}(z)+D_rE^L_{h_2}(z)],$$

$$G_6(z,z_1)=\frac{1}{2ikn_0B_r}[C_lE^L_{h_1}(z_1)+D_lE^L_{h_2}(z_1)]\times[C_rE^L_{h_1}(z)+D_rE^L_{h_2}(z)],$$

$$G_9(z,z_1)=\frac{1}{2ikn_0}\exp[ikn_0(z_1-L)]\{\exp[-ikn_0(z-L)]+A_r\exp[ikn_0(z-L)]\}.$$

In the above expressions $E^L_{h_1}$ and $E^L_{h_2}$ are the linearly independent solutions to the homogeneous equation (2) in the region $0<z<L$, which must be known in order to make practical use of the Green's function. All constants are directly related to the scattering solutions $E_l(z)$ and $E_r(z)$ of the homogeneous equation (2) for a plane wave falling on the structure from the regions $z<0$ and $z>L$, respectively (see Ref. [16]),

$$E_l(z)=\begin{cases}\exp(ikn_0z)+A_l\exp(-ikn_0z), & z<0\\ C_lE^L_{h_1}(z)+D_lE^L_{h_2}(z), & 0<z<L\\ B_l\exp[ikn_0(z-L)], & z>L\end{cases} \tag{7}$$

and

$$E_r(z)=\begin{cases}B_r\exp(-ikn_0z), & z<0\\ C_rE^L_{h_1}(z)+D_rE^L_{h_2}(z), & 0<z<L\\ \exp[-ikn_0(z-L)]+A_r\exp[ikn_0(z-L)], & z>L.\end{cases} \tag{8}$$

They can be found from the continuity conditions if the fundamental solutions $E^L_{h_1}$ and $E^L_{h_2}$ are known. The boundary conditions require that $A_r^*=A_l$ and $B_l=B_r$ for any $E^L_{h_1}$ and $E^L_{h_2}$ [17].

Suppose we have found the fundamental solutions $E^L_{h_1}$ and $E^L_{h_2}$ by some means, either analytically or numerically. Now let us consider the same dielectric structure but with an embedded defect of refractive index $n_\sigma(z)$ and with finite width $\sigma=z''-z'$ between $0<z<L$. The one-dimensional Helmholtz equation (2) for such a refractive index profile $n(z)$ can be represented in the form of the inhomogeneous equation

$$\frac{d^2E(z)}{dz^2}+k^2n_h^2(z)E(z)=\begin{cases}0, & z<z',z>z''\\ -k^2[n_\sigma^2-n_h^2(z)]E(z), & z'<z<z''\end{cases} \tag{9}$$

with the corresponding homogeneous equation and its solution $E_h(z)$ describing wave propagation through the structure without the defect (1). One can see that for the wave $\exp(ikn_0z)$ incoming from the region $z<0$ $E_h(z)=E_l(z)$ with $E_l(z)$ from Eq. (7). The solution $E(z)$ of the inhomogeneous equation (9) itself also has the form of expression (7) but with a different fundamental system of solutions $E^L_1(z)$, $E^L_2(z)$ inside the structure and with different constants, which we denote by A, B, C, and D.

Introducing the parameter $\mu(z)=-k^2[n_\sigma^2(z)-n_h^2(z)]$, let us change the inhomogeneous differential equation (9) to the equivalent integral Lippmann-Schwinger equation [13], which in the region $z<0$ has the form

$$\exp(ikn_0z)+A\exp(-ikn_0z)=\exp(ikn_0z)+A_l\exp(-ikn_0z)+\int_{z'}^{z''}G(z,z_1)\mu(z_1)E^L(z_1)dz_1. \tag{10}$$

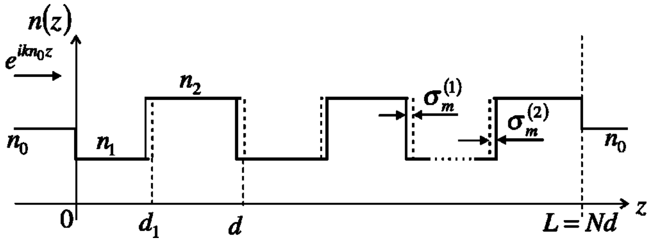

FIG. 1. Two-layered periodic dielectric structure with fluctuations in layer thicknesses: dashed line, ideal structure (no fluctuations in layer thicknesses); solid line, real structure (fluctuations in layer thicknesses with Gaussian distribution).

This integral equation cannot be used directly to find the reflection coefficient A because under the integral we have the unknown function $E^L(z_1)=CE_1^L(z_1)+DE_2^L(z_1)$, which is a solution of Eq. (9) in the region $0<z<L$. However, it is possible to develop an approximation scheme known as the Neumann series. The core idea of the scheme for our problem is to use the homogeneous solution $E_h^L(z_1)=C_lE_{h_1}^L(z_1)+D_lE_{h_2}^L(z_1)$ as an initial approximation for the inhomogeneous solution $E^{(L)}(z_1)$ under the integral, and then iterate to find higher order corrections. As a result, we can express the reflection coefficient A in terms of the Neumann series

$$A=A_l+A^{(1)}+A^{(2)}+\cdots+A^{(j)}+\cdots, \tag{11}$$

where the first term of the series A_l is simply the amplitude reflection coefficient for the structure (1) (no defects), the second term

$$A^{(1)}=\int_{z'}^{z''}G(0,z_1)\mu(z_1)[C_lE_{h_1}^L(z_1)+D_lE_{h_2}^L(z_1)]dz_1 \tag{12}$$

is the first-order correction to the reflection coefficient, and an arbitrary term is given by

$$A^{(j)}=\int_{z'}^{z''}\cdots\int_{z'}^{z''}G(0,z_j)\mu(z_j)\cdots G(z_2,z_1)\mu(z_1)\times[C_lE_{h_1}^L(z_1)+D_lE_{h_2}^L(z_1)]dz_1dz_2\cdots dz_j. \tag{13}$$

III. APPLICATION TO TWO-LAYERED PERIODIC STRUCTURES

In this section we apply the general solution to a finite two-layered periodic dielectric structure with random fluctuations in the width of the layers due, for example, to inhomogeneous growing conditions. Let the number of periods be N and the thicknesses of the ideal layers be d_1,d_2, i.e., for the ideal structure $L=Nd$, where $d=d_1+d_2$ is the period of the structure, and the refractive index is

$$n_L(z)=\begin{cases} n_1, & (m-1)d<z<md_1 \\ n_2, & md_1<z<md, \end{cases} \tag{14}$$

where $m=1,2,\ldots,N$ is the number of the current period (see Fig. 1).

In order to describe the random fluctuations, we will use a normal distribution model with mean 0 and standard deviation δr_1 for the fluctuations in the width of the layers with $n(z)=n_1$, and standard deviation δr_2 for the fluctuations in the width of the layers with $n(z)=n_2$. As a result, the actual widths of the layers with the refractive indexes n_1 and n_2 of an arbitrary period m are $d_1+\delta d_{1m}$ and $d_2+\delta d_{2m}$, where δd_{1m} and δd_{2m} are either positive or negative random numbers from the above normal distributions, as shown in Fig. 1. Therefore, it is convenient to introduce two groups of embedded defects inside the structure. The first group consists of N defects with thickness, boundary points, and refractive index of the defect in the mth period determined by the formulas

$$\sigma_m^{(1)}=\sum_{j=1}^{m}\delta d_{1j}+\sum_{j=1}^{m-1}\delta d_{2j},$$
$$z'=z_m^{(1)}\equiv(m-1)d+d_1, \tag{15}$$
$$z''=z_m^{(1)}+\sigma_m^{(1)},$$
$$n_\sigma(z)=\begin{cases} n_1 & \text{if } \sigma_m^{(1)}>0 \\ n_2 & \text{if } \sigma_m^{(1)}<0. \end{cases}$$

As a result, the perturbation potential $\mu(z)$ of the defect of this group in the mth period takes the form

$$\mu_m^{(1)}=\begin{cases} -k^2(n_1^2-n_2^2) & \text{if } \sigma_m^{(1)}>0 \\ -k^2(n_2^2-n_1^2) & \text{if } \sigma_m^{(1)}<0. \end{cases} \tag{16}$$

The second group consists of $N-1$ defects with thickness, boundary points, and refraction index of the defect on the boundary between the mth and $m+1$th periods determining by the formulas

$$\sigma_m^{(2)}=\sum_{j=1}^{m}\delta d_{1j}+\sum_{j=1}^{m}\delta d_{2j},$$
$$z'=z_m^{(2)}\equiv md, \tag{17}$$
$$z''=z_m^{(2)}+\sigma_m^{(2)},$$
$$n_\sigma(z)=\begin{cases} n_2 & \text{if } \sigma_m^{(2)}>0 \\ n_1 & \text{if } \sigma_m^{(2)}<0. \end{cases}$$

As a result, the perturbation potential $\mu(z)$ for each defect of this group takes the form

$$\mu_m^{(2)}=\begin{cases} -k^2(n_2^2-n_1^2) & \text{if } \sigma_m^{(2)}>0 \\ -k^2(n_1^2-n_2^2) & \text{if } \sigma_m^{(2)}<0. \end{cases} \tag{18}$$

Now let us find the amplitude reflection coefficient A of this structure for the case of a plane wave $\exp(ikn_0z)$ incoming from the region $z<0$. For the ideal periodic profile the homogeneous equation (2) in the interval $0<z<L$ is the Hill equation, and, according to the Floquet theorem, the solutions $E^L_{h_1}(z)$ and $E^L_{h_2}(z)$ can be represented as [18]

$$E^L_{h_1}(z)=F_1(z)\exp(i\xi z),\quad E^L_{h_2}(z)=F_2(z)\exp(-i\xi z), \tag{19}$$

where $F_1(z)$ and $F_2(z)$ are periodic functions of z with the period d, and ξ is a characteristic Lyapunov constant. In the specific case of the two-layered periodicity, $E^L_{h_1}(z)$ and $E^L_{h_2}(z)$ can be written in the layers with $n(z)=n_1$ as [1,19]

$$E^{n_1}_{h_1}(z)=\sin\{kn_1[z-(m-1)d]-\tfrac{1}{2}kn_1d_1+\varphi\}e^{i\xi(m-1)}, \tag{20}$$

$$E^{n_1}_{h_2}(z)=\sin\{kn_1[z-(m-1)d]-\tfrac{1}{2}kn_1d_1-\varphi\}e^{-i\xi(m-1)},$$

and in the layers with $n(z)=n_2$ as

$$E^{n_2}_{h_1}(z)=\tfrac{1}{2}\left(1+\frac{n_1}{n_2}\right)\sin[kn_2(z-md)-\tfrac{1}{2}kn_1d_1+\varphi]e^{i\xi m} -\tfrac{1}{2}\left(1-\frac{n_1}{n_2}\right)\sin[kn_2(z-md)+\tfrac{1}{2}kn_1d_1-\varphi]e^{i\xi m}, \tag{21}$$

$$E^{n_2}_{h_2}(z)=\tfrac{1}{2}\left(1+\frac{n_1}{n_2}\right)\sin[kn_2(z-md)-\tfrac{1}{2}kn_1d_1-\varphi]e^{-i\xi m} -\tfrac{1}{2}\left(1-\frac{n_1}{n_2}\right)\sin[kn_2(z-md)+\tfrac{1}{2}kn_1d_1+\varphi]e^{-i\xi m}.$$

The complex phase ϕ and so-called multiplicator $\rho := \exp(i\xi)$ can be expressed as

$$\varphi=\tfrac{1}{2}\arccos\left[\frac{a^{-1}\sin\Omega+a\sin\Delta}{2\sin(kn_2d_2)}\right], \tag{22}$$

$$\rho=\frac{\cos(\Omega)-a^2\cos(\Delta)-\mathrm{sgn}[a\sin(kn_2d_2)]\sqrt{[\cos\Omega-a^2\cos\Delta]^2-(1-a^2)^2}}{1-a^2}, \tag{23}$$

where

$$a=\frac{n_2-n_1}{n_2+n_1},\quad \Omega=k(n_2d_2+n_1d_1),\quad \Delta=k(n_2d_2-n_1d_1). \tag{24}$$

Using the boundary conditions at the points $z=0$ and $z=Nd$, we can define either analytically or numerically all constants A_l, B_l, C_l, D_l, A_r, B_r, C_r, and D_r of the scattering solutions (7) and (8). For example, the constant A_l, which is the amplitude reflection coefficient of the ideal structure for the wave $\exp(ikn_0z)$ incoming from the region $z<0$, has the analytical form

$$A_l=\frac{[(n_1^2-n_0^2)\cos(2\phi)+(n_1^2+n_0^2)\cos(kn_1d_1)+i2n_1n_0\sin(kn_1d_1)]\sin(\xi N)}{2n_1n_0\sin(2\phi)\cos(\xi N)-[(n_1^2-n_0^2)\cos(kn_1d_1)+(n_1^2+n_0^2)\cos(2\phi)]\sin(\xi N)}. \tag{25}$$

As a result, a knowledge of all these constants allows us to find the Green's function $G(z,z_1)$ for the finite ideal two-layered periodic structure in all 12 parts (5) of the plane (z,z_1).

Finally, the net contribution of the fluctuations to the first-order correction term (12) for the reflection coefficient can be written as

$$A^{(1)}=\sum_{m=1}^{N}\mu_m^{(1)}\int_{z_m^{(1)}}^{z_m^{(1)}+\sigma_m^{(1)}}dz_1 \times\begin{cases}G_6^{n_2}(0,z_1)E_h^{n_2}(z_1) & \text{if}\ \ \sigma_m^{(1)}>0\\ G_6^{n_1}(0,z_1)E_h^{n_1}(z_1) & \text{if}\ \ \sigma_m^{(1)}<0\end{cases}$$
$$+\sum_{m=1}^{N-1}\mu_m^{(2)}\int_{z_m^{(2)}}^{z_m^{(2)}+\sigma_m^{(2)}}dz_1 \times\begin{cases}G_6^{n_1}(0,z_1)E_h^{n_1}(z_1) & \text{if}\ \ \sigma_m^{(2)}>0\\ G_6^{n_2}(0,z_1)E_h^{n_2}(z_1) & \text{if}\ \ \sigma_m^{(2)}<0,\end{cases} \tag{26}$$

where

$$G_6^{n_{1,2}}(0,z_1)=\frac{1}{2ikn_0}[C_lE_{h_1}^{n_{1,2}}(z_1)+D_lE_{h_2}^{n_{1,2}}(z_1)] \tag{27}$$

and

$$E_h^{n_{1,2}}(z_1)=C_lE_{h_1}^{n_{1,2}}(z_1)+D_lE_{h_2}^{n_{1,2}}(z_1). \tag{28}$$

Using Eqs. (27) and (28), we can express $A^{(1)}$ in the more explicit form

$$A^{(1)}=\sum_{m=1}^{N}\frac{\mu_m^{(1)}}{2ikn_0}\int_{z_m^{(1)}}^{z_m^{(1)}+\sigma_m^{(1)}}dz_1\times\begin{cases}[C_lE_{h_1}^{n_2}(z_1)+D_lE_{h_2}^{n_2}(z_1)]^2 & \text{if } \sigma_m^{(1)}>0\\ [C_lE_{h_1}^{n_1}(z_1)+D_lE_{h_2}^{n_1}(z_1)]^2 & \text{if } \sigma_m^{(1)}<0\end{cases}$$
$$+\sum_{m=1}^{N-1}\frac{\mu_m^{(2)}}{2ikn_0}\int_{z_m^{(2)}}^{z_m^{(2)}+\sigma_m^{(2)}}dz_1\times\begin{cases}[C_lE_{h_1}^{n_1}(z_1)+D_lE_{h_2}^{n_1}(z_1)]^2 & \text{if } \sigma_m^{(2)}>0\\ [C_lE_{h_1}^{n_2}(z_1)+D_lE_{h_2}^{n_2}(z_1)]^2 & \text{if } \sigma_m^{(2)}<0.\end{cases} \tag{29}$$

All four kinds of integral in the above expression can be evaluated analytically without any difficulties, since the functions under the integral are simply superposition of the sinusoidal functions (20) and (21). However, the final answer is very cumbersome and we do not present it here in detail. Instead, in the next section, we consider the numerical application of Eq. (29) to two-layered periodic dielectric structures with the specific parameters suitable for the construction of optical switching systems. It should be noted that we can obtain any other correction term $A^{(j)}$ by applying formula (13).

IV. RESULTS AND DISCUSSION

As mentioned in the Introduction, the main idea for a proposed optical switch is to vary the material parameters of the two-layered periodic dielectric medium so as to change significantly (up to 80%) the reflection coefficient for incident electromagnetic waves with a specific wavelength λ by the application of an elastic stress of reasonable size. The proposal differs from existing acousto-optic filters and switches in that it is the thicknesses of the basic layers d_1 and d_2 that are changed, rather than their indices of refraction n_1 and n_2.

Several requirements must be met in the choice of materials for the practical realization of such a switch. First, the reflection coefficient (as a function of λ) must have a well defined structure of forbidden regions, where the reflection coefficient almost reaches unity, and allowed regions, where the reflection coefficient drops to nearly zero. Second, the medium should be constructed from alternating layers of materials with low Young's modulus in one layer and high compressive yield strength in both in order to produce a differential change in the thicknesses d_1 and d_2 of the basic layers. Third, the parameters should be chosen to minimize the influence of unavoidable random fluctuations δd_{1m} and δd_{m2} on the performance characteristics of the switch due to imperfect growing conditions. The Green's function obtained here is particularly useful in studying the influence of random fluctuations in order to satisfy this last requirement.

There are two ways to satisfy the first requirement. A well defined structure of allowed and forbidden regions can be obtained either by using a rather large number N of layers consisting of alternating materials with close indices of refraction [1,19], or by using a small number of layers, but with materials having large differences in refractive index. The second option is preferable because it simultaneously helps to satisfy the third requirement—the smaller the number of layers, the smaller the number of terms in the sum (29) over random fluctuations in the calculation of corrections to the reflection coefficient. As for the second requirement, the most suitable material is a range of polymers with high optical transparency. Note that almost all optically transparent polymers have refractive indices in the range 1.33 to 1.50. Consequently, we cannot construct the operating medium from alternating layers of two polymers if we want to keep the number of layers small and still have a well defined structure of allowed and forbidden regions for the reflection coefficient. We are therefore forced to choose the second material to be a glass with high refractive index $n>2$. Such two-layered periodic structures consisting of alternating polymer/glass layers with a large difference in refractive indices have recently attracted considerable interest for other possible applications [20].

Let us consider a concrete example of such a structure. Consider a two-layered periodic structure consisting of $N=5$ periods of fluorinated ethylene propylene (FEP) polymer with $n_1=1.344$, $d_1=3.88$ μm, Young's modulus $E_1=380$ N/mm^2, and Poisson's ratio $\sigma_1=0.48$ alternating with a chalcogenide glass based on GaS_3-La_2S_3 with $n_2=2.4$, $d_2=2.17$ μm, and $E_2=78.4\times10^3$ N/mm^2 [21]. The structure is assumed to be surrounded by the same chalcogenide glass.

The two curves in Fig. 2 represent the dependence of the reflection coefficient on the wavelength λ of the incident radiation. The dashed curve characterizes reflection from an ideal structure corresponding to the formula $A=A_l$ with A_l from Eq. (25). The solid curve characterizes the reflection from a real structure with random fluctuations in layer thickness, calculated at the first level of approximation, using the formula

$$A=A_l+A^{(1)} \tag{30}$$

with $A^{(1)}$ obtained from a numerical evaluation of formula (29). For the standard deviations $\delta r_{1,2}$ of the normal distribution we use 0.25% of the corresponding width of the ideal layers. This figure is in accordance with the actual precision of the procedure used for the preparation of thin films of polymers [21]. We can see that for $\lambda=0.633$ μm (He-Ne laser) the reflection coefficient almost equals 100% (no transmission) for both ideal and real structures. Note that, according to the perturbation theory, we can use formula (30) only if the numerical value of $A^{(1)}$ obtained from Eq. (29) is less than numerical value of A_l obtained from Eq. (25), which is true for the two-layered periodic structure above.

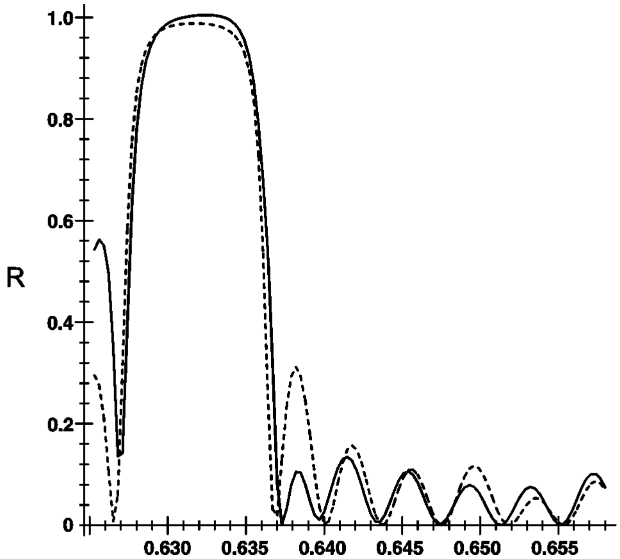

FIG. 2. Dependence of R on λ_0 for $N=5$, $n_0=2.4$, $n_1=1.344$, $n_2=2.4$, $d_1=3.88$ μm, $d_2=2.17$ μm; dashed line, ideal structure (no fluctuations in layer thicknesses); solid line, real structure (dispersion of fluctuations in layer thicknesses is 0.25%).

If we apply an external compressive stress p to the boundaries of the structure only in the direction of the periodicity z (such a deformation is called a simple compression), the homogeneous decrease in the thickness of each basic layer is determined by [22]

$$\delta d_{1,2}=\frac{p_{1,2}}{E_{1,2}}d_{1,2}, \tag{31}$$

where p is the applied stress in the z direction.

In order to evaluate the variations in the refractive indices $\delta n_{1,2}$ under the applied stress, we use as a starting point the Lorentz-Lorenz equation, which relates the index of refraction n to the molecular polarizability α for isotropic and cubic materials,

$$\frac{n^2-1}{n^2+2}=\frac{4\pi}{3}\frac{N_A\rho}{M}\alpha, \tag{32}$$

where N_A is Avogadro's number, M is the molecular weight, and $N_A\rho/M$ is the number of molecules per unit volume. If we assume that the polarizability α changes with changes in the density of material ρ as $\delta\alpha/\alpha=\Lambda_0\delta\rho/\rho$ [23], by straightforward differentiation of the expression (32) we obtain the relation

$$\frac{\delta n}{\delta\rho}=\frac{(n^2-1)(n^2+2)}{6n\rho}(1-\Lambda_0), \tag{33}$$

where Λ_0 is the phenomenological strain polarizability constant. If, for example, Λ_0 is equal to zero, the changes in the refractive index n are produced only by changes in the density of the material ρ. In most cases, however, Λ_0 is not zero. For the typical polymer, such as polysterene, $\Lambda_0=0.4\pm0.1$ [24].

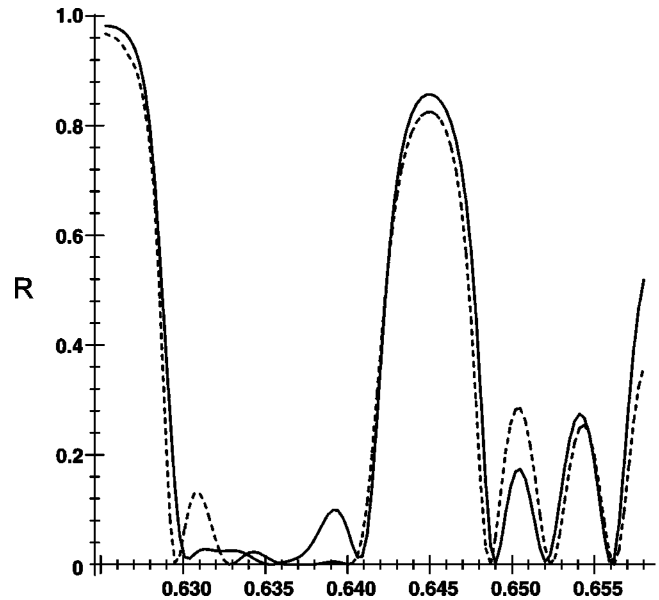

FIG. 3. Dependence of R on λ_0 for $N=5$, $n_0=2.4$, $n_1=1.346$, $n_2=2.4$, $d_1=3.79$ μm, $d_2=2.17$ μm (the previous structure under applied stress $p=9$ N/mm^2); dashed line, ideal structure (no fluctuations in layer thicknesses); solid line, real structure (dispersion of fluctuations in layer thicknesses is 0.25%).

For our case of a simple compression, we can express the relative changes in densities of our materials as $\delta\rho_{1,2}/\rho_{1,2}=p(1-2\sigma_{1,2})/E_{1,2}$. As a result, the variations in the values of the refractive indexes can be written in the form

$$\delta n_{1,2}=\frac{(n_{1,2}^2-1)(n_{1,2}^2+2)}{6n_{1,2}}\frac{p(1-2\sigma_{1,2})}{E_{1,2}}(1-\Lambda_0). \tag{34}$$

For the relatively small stress $p=9$ N/mm^2, which is far enough from the compressive yield point (lower limit of plastic deformation) of FEP (15 N/mm^2), we can obviously neglect the variations in the parameters of the glass n_2 and d_2 because of its high value of Young's modulus E_2 in comparison with the Young's modulus E_1 for the FEP. As for variations in the parameters of the FEP layers, using Eqs. (34) and (31), we obtain $\delta n_1=0.002$ and $\delta d_1=0.09\mu$m. The results are summarized in Fig. 3. The dashed line again represents the reflective coefficient dependence for an ideal compressed structure, i.e., for a structure with refraction indexes of the basic layers $n_1+\delta n_1, n_2$ and the widths of the basic layers $d_1+\delta d_1, d_2$. The solid line represents the first-order approximation to the reflection coefficient dependence for the real compressed structure, i.e., for the structure with the width of the basic layers $d_1+\delta d_1+\delta d_{1m}$, $d_2+\delta d_{2m}$. We can see now that for $\lambda=0.633$ μm R for both structures is less than 10%, i.e., we have almost full transmission.

Therefore, the reflection and transmission coefficients are not critically sensitive to fluctuations of 0.25% in the thicknesses of the basic layers of a two-layered periodic dielectric structure with a small number of periods $(N=5)$. As a result, a two-layered periodic dielectric structure with such parameters seems suitable for the construction of optical switching devices based on the effect of a decrease of layer thicknesses under applied stress.

V. CONCLUSION

We have applied Green's function theory to the problem of the propagation of electromagnetic waves through a two-layered periodic dielectric structure with random Gaussian fluctuations in the thicknesses of the basic layers. The main idea of the method is to build an exact analytical Green's function for a two-layered periodic structure using its symmetry properties. Then, with the aid of the Lippmann-Schwinger equation, we found the first-order correction to the reflection coefficient of an ideal structure due to fluctuations in layer thicknesses. We have shown that for an electromagnetic wave whose wavelength is several times the period of the structure (this ratio corresponds, for example, to the optical range of waves and a structure with the size of basic layers of 2–5 μm) fluctuations in layer thicknesses of the order 0.25% do not significantly change the reflection and transmission coefficients for either unstressed or stressed structures if the number of periods does not exceed $N=6$. As a result, an optical switching technique based on the effect of the compression of a two-layered periodic structure with a relatively small number of periods and with high optical modulation, i.e., consisting of materials having a large difference in refractive indices, is feasible despite the influence of unavoidable fluctuations in layer thicknesses.

The analysis presented here is just one example of a wide range of problems that can be solved once the Green's function for the basic periodic structure is known in analytical form. Other applications will be discussed in future publications.

ACKNOWLEDGMENT

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